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On asymptotic finite-time ruin probabilities of a new bidimensional risk model with constant interest force and dependent claims

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Abstract

Consider a new continuous-time bidimensional renewal risk model with constant force of interest, in which every kind of business is assumed to pay two classes of claims called the first and second ones, respectively. Suppose that the first class of claim vectors form a sequence of independent and identically distributed random vectors following a general dependence structure which share a common renewal counting process, and the second class of claim vectors, independent of the first class of claim vectors, constitute another sequence of independent and identically distributed random vectors which arrive according to two different renewal counting process. For such a model, when the claims are assumed to be subexponential or belong to the intersection of long-tailed and dominatedly varying-tailed class, some asymptotic formulas on finite-time ruin probabilities are derived. The obtained results substantially extend some existing ones in the literature.

Introduction

Introduce a new continuous-time bidimensional renewal risk model. Its surplus process $\{\left(R_1(x,t),R_2(y,t)\right)^{\top};t\geq 0\}$ is described as

$$\begin{pmatrix} R_1(x,t)\\ R_2(y,t)\end{pmatrix} = e^{rt}\begin{pmatrix} x\\ y\end{pmatrix} + \begin{pmatrix} \int_{0-}^{t} e^{r(t-s)}C_1(ds)\\ \int_{0-}^{t} e^{r(t-s)}C_2(ds)\end{pmatrix} - \begin{pmatrix} U_{01}(t)\\ U_{02}(t)\end{pmatrix} \tag{1}$$

with

$$U_{0k}(t)=\sum_{i=1}^{N_0(t)} X_i^{(k)}e^{r(t-\sigma_i^{(0)})}+\sum_{j=1}^{N_k(t)} Y_j^{(k)}e^{r(t-\sigma_j^{(k)})},\ k=1,2,$$

where $\{\left(R_1(x,t),R_2(y,t)\right)^{\top};t\geq 0\}$ denotes the bidimensional surplus process, $(x,y)^{\top}$ the vector of initial surplus, $r\geq 0$ the constant force of interest, $\{\left(C_1(t),C_2(t)\right)^{\top};t\geq 0\}$ the bidimensional premium process, Additionally, for $k=1,2$, $\{(X_i^{(k)},Y_j^{(k)})^{\top},i\geq 1,j\geq 1\}$ are assumed to be two classes of claim sizes from the kth kind of business, and $\{N_i(t),t\geq 0\},i=0,1,2,$ to be three independent renewal counting processes with corresponding arrival times $\{\sigma_j^{(i)},j\geq 1\},i=0,1,2$. Here we call $\{(X_i^{(1)},X_j^{(2)})^{\top},i\geq 1,j\geq 1\}$ the first class of claim vectors and $\{(Y_i^{(1)},Y_j^{(2)})^{\top},i\geq 1,j\geq 1\}$ the second class of claim vectors. Assume that the first class of claim vectors share a common claim-arrival process $\{N_0(t),t\geq 0\}$ and the second class of claim vectors arrive according to two different renewal counting processes $\{N_1(t),t\geq 0\}$ and $\{N_2(t),t\geq 0\}$, respectively. The mean function of $\{N_k(t),t\geq 0\}$ is denoted by $\lambda_k(t)<\infty$ and further define $\Lambda_k=\{t>0,\lambda_k(t)>0\}=\{t>0,P(\sigma_1^{(k)}\leq t)>0\}$ for later use. We write $\Lambda=\bigcap_{k=0}^{2}\Lambda_k$.

For the convenience of later use, in what follows, for $k=1,2$, assume that the claim sizes $\{X_i^{(k)},i\geq 1\}$ and $\{Y_i^{(k)},i\geq 1\}$ are independent and identically distributed (i.i.d.) nonnegative random variables (r.v.s) with generic r.v.s $X^{(k)}$ and $Y^{(k)}$ distributed by F_k and G_k, respectively. Moreover, suppose that $\{(X_i^{(1)},X_j^{(2)}),i\geq 1,j\geq 1\}$, $\{Y_i^{(1)},i\geq 1\}$ and $\{Y_i^{(2)},i\geq 1\}$ are mutually independent, but the random pair $(X^{(1)},X^{(2)})$ follows a general dependence structure proposed in [2].

Assumption 1. There exist a positive constant ρ such that

$$P(X^{(1)}>x,X^{(2)}>y)\sim\rho\overline{F_1}(x)\overline{F_2}(y), \tag{2}$$

where the symbol $\sim$ means that the quotient of both sides tends to 1 as $x,y\to\infty$. As stated in [2], it is easy to check that many commonly used copulas, including the bivariate Farlie-Gumbel-Morgenstern (FGM) one, satisfy Assumption 1. For the above nonstandard bidimensional renewal risk models, we define the following three types of finite-time ruin times by convention. For any fixed $t>0$, first define the ruin times of the two marginal processes within a finite time $t>0$ as

$$\tau^{(1)}(x)=\inf\{t:R_1(x,t)<0\}\quad\text{and}\quad\tau^{(2)}(y)=\inf\{t:R_2(y,t)<0\}.$$

Further define

$$\tau_{\text{and}}(x,y)=\max\{\tau^{(1)}(x),\tau^{(2)}(y)\},\quad \tau_{\text{or}}(x,y)=\min\{\tau^{(1)}(x),\tau^{(2)}(y)\}$$

and

$$\tau_{\text{sum}}(x,y)=\inf\{t>0:R_1(x,t)+R_2(y,t)<0\}.$$

Then the corresponding finite-time ruin probabilities are respectively defined by

$$\begin{aligned}\psi_{\text{and}}(x,y;t) &= P\left(\tau_{\text{and}}(x,y)\leq t|R_1(x,0)=x,R_2(y,0)=y\right)\\ &= P\left(\left(\inf_{0\leq s\leq t}R_1(x,s)<0\right)\bigcap\left(\inf_{0\leq s\leq t}R_2(y,s)<0\right)\Big|R_1(x,0)=x,R_2(y,0)=y\right),\end{aligned}$$

which represents both of the surplus processes go below zero in the finite time;

$$\begin{aligned}\psi_{\text{or}}(x,y;t) &= P\left(\tau_{\text{or}}(x,y)\leq t|R_1(x,0)=x,R_2(y,0)=y\right)\\ &= P\left(\left(\inf_{0\leq s\leq t}R_1(x,s)<0\right)\bigcup\left(\inf_{0\leq s\leq t}R_2(y,s)<0\right)\Big|R_1(x,0)=x,R_2(y,0)=y\right),\end{aligned}$$

which represents at least one of the surplus processes goes below zero in the finite time and

$$\begin{aligned}\psi_{\text{sum}}(x,y;t) &= P\left(\tau_{\text{sum}}(x,y)\leq t|R_1(x,0)=x,R_2(y,0)=y\right)\\ &= P\left(\inf_{0\leq s\leq t}\left(R_1(x,s)+R_2(y,s)\right)<0\Big|R_1(x,0)=x,R_2(y,0)=y\right),\end{aligned}$$

which represents the sum of two surplus processes goes below zero in the finite time. In addition, in some existing literature, another type of finite-time ruin probability, denote by $\psi_{\text{sim}}(x,y;t)$, is also discussed; see, for instance, [3] (corresponds to $\psi(x,y;T)$ in his paper). Indeed, $\psi_{\text{sim}}(x,y;t)$ is defined as

$$\psi_{\text{sim}}(x,y;t)=P\left(\inf_{0\leq s\leq t}\left(\max\{R_1(x,s),R_2(y,s)\}\right)<0\Big|R_1(x,0)=x,R_2(y,0)=y\right),$$

which represents both of the surplus processes go below zero in the finite time simultaneously.

Main results

In this section, let us introduce our main results. For simplicity, we first use the following notations. Let

$$\begin{aligned}\varphi_\rho(x,y;t) &= \iint_{u,v>0,u+v\leq t}\left(\overline{F_1}(xe^{ru})\overline{F_2}(ye^{r(u+v)})+\overline{F_1}(xe^{r(u+v)})\overline{F_2}(ye^{rv})\right)\lambda_0(du)\lambda_0(dv)\\ &\quad+\rho\int_{0-}^{t}\overline{F_1}(xe^{rs})\overline{F_2}(ye^{rs})\lambda_0(ds),\end{aligned}$$

and for $i=1,2$,

$$\phi_{F_i}^{(0)}(x;t)=\int_{0-}^{t}\overline{F_i}(xe^{ru})\lambda_0(du),\quad \phi_{G_i}^{(i)}(x;t)=\int_{0-}^{t}\overline{G_i}(xe^{rv})\lambda_i(dv).$$

Theorem 1. *Consider the new bidimensional renewal risk model introduced in Section 1 with* $r\geq 0$. *If* $F_1,F_2,G_1,G_2\in\mathscr{S}$, $\overline{F_i}(x)\asymp\overline{G_i}(x),i=1,2$ *and Assumption 1 holds, then, for each* $t\in\Lambda$, *it holds that*

$$\begin{aligned}\psi_{\text{and}}(x,y;t) &\sim \psi_{\text{sim}}(x,y;t)\\ &\sim \varphi_\rho(x,y;t)+\phi_{F_1}^{(0)}(x;t)\phi_{G_2}^{(2)}(y;t)+\phi_{G_1}^{(1)}(x;t)\phi_{F_2}^{(0)}(y;t)+\phi_{G_1}^{(1)}(x;t)\phi_{G_2}^{(2)}(y;t),\end{aligned}$$

and

$$\psi_{\text{or}}(x,y;t)\sim\phi_{F_1}^{(0)}(x;t)+\phi_{F_2}^{(0)}(y;t)+\phi_{G_1}^{(1)}(x;t)+\phi_{G_2}^{(2)}(y;t).$$

Remark 1. Apparently, if we assume that both $N_1(t)$ and $N_2(t)$ equal to zero almost surely, then it follows from Theorem 1 that

$$\psi_{\text{sim}}(x,y;t)\sim\varphi_\rho(x,y;t),$$

which coincides with Theorem 1.1 of [2].

Theorem 2. *Consider the new bidimensional renewal risk model introduced in Section 1 with* $r\geq 0$. *If* $F_1,F_2\in\mathscr{D}\cap\mathscr{L}$, $G_1,G_2\in\mathscr{S}$, $\overline{F_i}(x)\asymp\overline{G_i}(x),i=1,2$ *and Assumption 1 holds, then, for each* $t\in\Lambda$, *we have that*

$$\psi_{\text{sum}}(x,y;t)\sim\phi_{F_1}^{(0)}(x+y;t)+\phi_{F_2}^{(0)}(x+y;t)+\phi_{G_1}^{(1)}(x+y;t)+\phi_{G_2}^{(2)}(x+y;t).$$

Remark 2. Particularly, if we assume that the claim vector (X,Y) consists of the independent components, then our Theorem 2 reduces to Theorem 1.1 of [1].

Further Research

Consider the results in Theorem 1 and Theorem 2 with $t=\infty$ under some stronger conditions.

References

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